Particle-number fluctuations near the critical point of nuclear matter

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Many works have been devoted to studying the properties of nuclear systems with strongly interacting particles. Thermodynamical behavior of the nuclear matter leads to liquid-gas first-order phase transition, which ends at the critical point, with special theoretical and experimental emphasis on finiteness of nuclear systems in multifragmentation reactions. The role and size of the effects of quantum statistics were studied [1] for van der Waals (vdW) and Skyrme (SLD) local density interparticle interactions for relatively low temperatures T, which is assumed to be smaller or of the order of 30 MeV and not too large particle number density ρ. In the present work [2] we apply the same analytical method for the equation of state [1] to analyze the particle number fluctuations ω near the critical point of nuclear matter with a focus on the finiteness of a sufficiently large average particle number $\langle N \rangle$ within the grand canonical ensemble.

Using the ideas of Smoluchowski and Einstein we use the expansion of the free energy $F(\rho)$ in powers of small deflections $\rho - n$, where $n = \langle \rho \rangle$ is a statistical average of the particle density ρ for symmetrical nuclear matter. The standard results for particle number fluctuations $\omega = T/K$, where K is the isothermal incompressibility, diverge at the critical point T_c, ρ_c of the nuclear matter because of $K \to 0$ for $T \to T_c$ and $\rho \to \rho_c$. They can be obtained for such a quadratic expansion of the free energy $F(\rho)$ on a finite distance from the critical point, defined by equations $(\partial P/\partial \rho)_T = 0$ and $(\partial^2 P/\partial \rho^2)_T = 0$, in the T, ρ plane for dominating second order terms with finite $K = (\partial P/\partial \rho)_T$. Here, $P = P(T, \rho)$ is the pressure of the equation of state. Taking into account the quantum statistical corrections at first order we derived the analytical expressions for such a fluctuation, $\omega = T/K$, near the critical point for the vdW and SLD effective interparticle interactions. This fluctuation is obtained as an explicit analytical functions of the temperature T, particle density ρ, particle's mass and their degeneracy in good agreement with more accurate numerical results; see, e.g., Ref. [3] for the SLD interaction case.

Within the same Smoluchowski and Einstein method, expanding $F(\rho)$ over powers of small differences $\rho - n$ up to fourth order terms and including the second order ones, for a finite second order derivative of the incompressibility, $K_2 = \partial^2 K/\partial \rho^2$ near the critical point $\rho = n_c$, we have derived analytically a simple expression for the generalized particle number fluctuations ω, in terms of the modified Bessel functions. We found the fluctuation ω as function of the parameter $\alpha \propto K^2 \langle N \rangle (n^2 T K_2)^{-1}$ depending on the particle number average $\langle N \rangle$. From this expression we obtained the two asymptotic limits. One of them is the Landau classical limit $\omega \to T/K$ for $\alpha \gg 1$, working well for infinite nuclear matter everywhere, except for a small range near the critical point. Another one is the Rowlinson and Tolpygo finite limit, approximately $\omega \to (6\langle N \rangle T (n^2 K_2)^{-1})^{(1/2)}$ to the critical point for $\alpha \ll 1$ at a finite particle-number average $\langle N \rangle$.

Fig. 1 shows the particle number fluctuations $\omega/\langle N\rangle$ as functions of the averaged density n at the critical temperature $T = T_c$ for different finite large values of $\langle N\rangle$. We clearly present divergence of the asymptote $\alpha \gg 1$ by dashed lines, and convergence to our generalized result (solid lines). The solid curves of a generalized approach have finite pronounced maxima near the critical density value $n \approx n_c$, in contrast to the Landau asymptotic formula. The SLD (with different set of parameters for $\gamma = 1/6$ and $\gamma = 1$) and vdW results for fluctuations are rather similar. The difference between the fluctuations $\omega/\langle N\rangle$ for the SLD and vdW forces are a slightly greater asymmetry of the curves for the vdW case with respect to the critical value of the particle number density, n_c, and a little larger values at the corresponding vdW maxima. Notice also that with increasing values of $\langle N\rangle$ the fluctuations $\omega/\langle N\rangle$ are decreasing to zero when $\langle N\rangle \to \infty$.

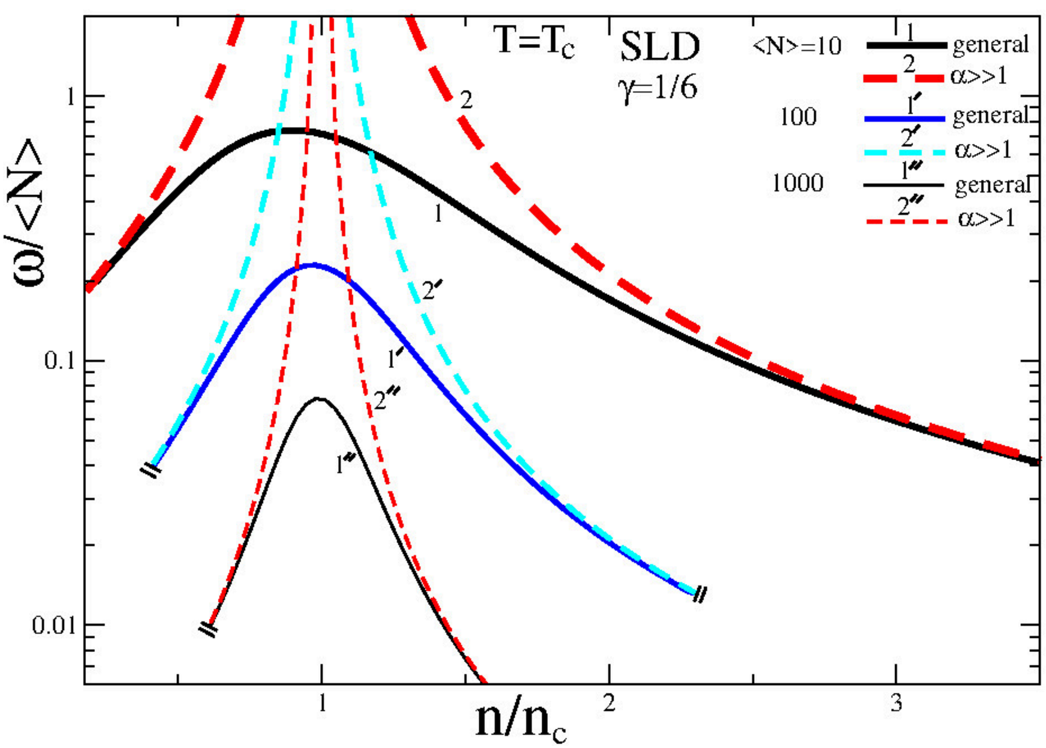

Fig. 1. Particle number fluctuations $\boldsymbol{\omega}$, divided by mean particle numbers $\langle \boldsymbol{N}\rangle$, at different typical values of $\langle \boldsymbol{N}\rangle$, are shown by solid lines as functions of the mean particle-number density $\boldsymbol{n}$ (in units of its critical value $\boldsymbol{n_c}$) at the critical temperature $\boldsymbol{T = T_c}$ for symmetric nucleon system with the Skyrme local-density effective interparticle interaction with some parameters $\boldsymbol{a = 1.167 GeV \cdot fm^3}$, $\boldsymbol{b = 1.475 GeV \cdot fm^{3+3\gamma}}$, and $\boldsymbol{\gamma = 1/6}$, for which one has [1,2] good agreement for the critical point with more accurate numerical calculations [3]. Dashed lines present the corresponding Landau asymptotes for $\boldsymbol{\alpha \gg 1}$.

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